Inverse Heat Conduction Study

Jordan Wall
12/16/2009
Inverse heat conduction is a class of problems where prescribed temperatures at a given location are used to determine time varying boundary conditions at another location.

Direct problems with prescribed boundary and initial conditions, studied previously, deal with generally stable solutions.

Inverse heat conduction problems on the other hand are not well posed and solutions tend to be sensitive to changes in input data.

This class of problems is dealt with exhaustively in the field of heat transfer when trying to match analytical models to test data.
PROBLEM

Determine heat flux and temperature on the plate front face on the basis of measured temperature transient on the insulated back surface.

Governing Differential Equation

\[ k \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t} \]

Boundary and Initial Conditions

\[ -k \frac{\partial T}{\partial x} = q(t) \quad @ \ x = L \]

\[ \frac{\partial T}{\partial x} = 0 \quad @ \ x = 0 \]

\[ T(x,0) = 20^\circ C \]
The inverse heat conduction method requires temperatures over time as input. In order to obtain input for the IHC Fortran code and also verify the final solution, a finite difference code was written to determine temperatures from a given heat flux input.

The temperatures output from the finite difference solution were then used as input for the IHC Fortran code.

The IHC Fortran code relies on Thaler’s data matching formula and its derivatives shown in Solving Direct and Inverse Heat Conduction Equations:

\[ y_i = y(t_i) = \frac{1}{693} (-53 f_i - 4 + 117 f_{i-2} + 162 f_{i-1} + 177 f_i + 162 f_{i+1} +
+117 f_{i+2} + 42 f_{i+3} - 63 f_{i+4}), \]
\[ y'_i = \frac{dy}{dt}_{t=i} = \frac{1}{1188 \Delta t} (86 f_{i-4} - 142 f_{i-3} - 193 f_{i-2} - 126 f_{i-1} - 126 f_{i+1} +
+193 f_{i+2} + 142 f_{i+3} - 86 f_{i+4}), \]
\[ y''_i = \frac{d^2y}{dt^2}_{t=i} = \frac{1}{462 (\Delta t)^2} (28 f_{i-4} + 7 f_{i-3} - 8 f_{i-2} - 17 f_{i-1} - 20 f_i -
-17 f_{i+1} - 8 f_{i+2} + 7 f_{i+3} + 28 f_{i+4}), \]
\[ y'''_i = \frac{d^3y}{dt^3}_{t=i} = \frac{1}{198 (\Delta t)^3} (-14 f_{i-4} + 7 f_{i-3} + 13 f_{i-2} + 9 f_{i-1} - 9 f_{i+1} -
-13 f_{i+2} - 7 f_{i+3} + 14 f_{i+4}), \]
RESULTS

The results show good comparison between the prescribed heat flux and the predicted heat flux.

Cold and hot side (with and without noise) temperature profile over time for a triangular heat flux input

Prescribed and calculated heat flux over time for a triangular input
RESULTS

The results show good comparison between the prescribed heat flux and the predicted heat flux except for the anticipated step change at 1500s.
CONCLUSIONS

The inverse heat conduction method can be tailored to many applications in industry.

This would be particularly useful in my field when trying to match measured thermal data in jet engines to analytical and finite element models.

This case study showed that the resulting heat flux from the inverse heat conduction Fortran code matched the prescribed input fairly well.

However, the cases study ran into problems when there was a large step change in heat flux.

RECOMMENDATIONS

For more accurate results, it would be advantageous to decrease the time step or add a slope test to the code which could modify the data matching formula when the slope nears infinity.
BACKUPS
Cold and hot side (with and without noise) temperature profile over time for a triangular heat flux input
Prescribed and calculated heat flux over time for a triangular input
Cold and hot side (with and without noise) temperature profile over time for a triangular heat flux input